

# Dynamical Properties of the $\sigma$ Meson<sup>1</sup>

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**Abstract.** Studies on the dynamical properties of the  $\sigma$  meson are reviewed and discussed. The important role of fundamental principles such as analyticity, unitarity and crossing symmetry played in the studies are stressed.

**Keywords:** Scalar meson, chiral symmetry, dispersion relations

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Remarkable progress has been made in recent few years in revealing unambiguously the existence of the light and broad resonance  $f_0(600)$ , [1] for which dispersive analyses play a crucial role. [2, 3] Nevertheless there still remains difficulties in understanding the nature of the  $f_0(600)$  or the  $\sigma$  meson at the fundamental (i.e., QCD) level, though efforts are started to make from lattice approach. [4]

There exist, however, at phenomenological level, many investigations on the dynamical property of the  $\sigma$  meson. Most of these studies are based upon  $\sigma$ -like models. [5] There are also attempts trying to interpret the  $f_0(600)$  as a dynamically generated resonance. [6, 7] Dispersive analyses are also made trying to understand the nature of  $\sigma$  from  $\gamma\gamma \rightarrow \pi\pi$  processes. [8]

A study on the property of the  $\sigma$  meson using lagrangian models encounter a problem: the large widths of the  $\sigma$  pole [3] and the  $\kappa$  pole [9, 10] indicate that there must be very strong interactions involved. Since we know little about how to do a calculation based on a lagrangian except making perturbation expansions, certain unitarization approximation has to be used.

An examination to the unitarized chiral perturbative amplitudes finds a light and broad pole on the complex  $s$  plane, in the  $I,J=0,0$  channel of  $\pi\pi$  scattering, which is identified as the  $\sigma$  pole. It is found that the  $N_c$  trajectory of the  $\sigma$  pole has a non-typical behavior as comparing with that of a normal resonance, e.g., a  $\rho$  pole. Hence it is argued that the  $\sigma$  is a dynamically generated resonance from a lagrangian without the  $\sigma$  degree of freedom. [6] This idea has been carefully examined[11, 12], and it is found that the  $[1,1]$  Padé approximation leads to a ' $\sigma$ ' pole falling back to the real  $s$  axis in the large  $N_c$  limit. A correct understanding on what does the  $[1,1]$  Padé approximant mean is obtained through these studies. The  $\sigma$  pole position can be obtained analytically from  $[1,1]$  Padé approximant in the large  $N_c$  and chiral limit. Meanwhile the same pole position can be found in a model *independent* way, but under two additional assumptions: 1)  $s$ -channel  $\sigma$  pole dominance, 2) neglecting the left hand cut. [11] We will carefully study what

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does the second assumption mean a while later in this talk. It is also pointed out that the bent structure of the  $\sigma$  pole trajectory with respect to  $N_c$  found in [1,1] Padé approximant is in qualitative agreement with what one finds in  $O(N_f)$   $\sigma$  model, hence suggesting a fundamental role of the light and broad resonance pole played at lagrangian level, even though it can be generated from certain dynamical approximations. [12] Here  $N_f$  means the number of flavors and one need not worry the lost of asymptotic freedom when  $N_f$  goes large. The real meaning of  $O(N_f)$  toy model is that, a large  $N_f$  enables one to neglect the cross channel dynamics legally and hence solve the model analytically.

One constructs, using analyticity, unitarity, and *partial wave* dispersion relations, a factorized form for elastic scattering  $S$  matrix element: [9]

$$S^{phy.} = \prod_i S^{R_i} \cdot S^{cut} , \quad (1)$$

where  $S^{R_i}$  denotes the  $i$ -th *second* sheet pole contribution and  $S^{cut}$  denotes the contribution from cuts or background. The information from higher sheet poles is hidden in the right hand integral which consists of one part of the total background contribution,

$$\begin{aligned} S^{cut} &= e^{2i\rho f(s)} , \\ f(s) &= \frac{s}{\pi} \int_L \frac{\text{Im}_L f(s')}{s'(s'-s)} + \frac{s}{\pi} \int_R \frac{\text{Im}_R f(s')}{s'(s'-s)} . \end{aligned} \quad (2)$$

The ‘left hand’ cut  $L = (-\infty, 0]$  for equal mass scatterings and may have a rather complicated structure for unequal mass scatterings. The right hand cut  $R$  starts from first *inelastic* threshold to positive infinity. Estimates on the ‘left’ cuts in various channels of  $\pi\pi$  and  $\pi K$  scatterings using  $\chi$ PT reveal a common feature: all the left cut contributions as defined in Eq. (2) are numerically found to be negative! This fact is actually crucial to establish the existence of the  $\sigma$  and  $\kappa$  pole in the present approach and also helps greatly in stabilizing the pole location in the data fit. It is interesting to notice that, there actually exists a correspondence of Eq. (1) in quantum mechanical scattering theory, obtained sixty years ago:[13]

$$S(k) = e^{-2ikR} \prod_1^\infty \frac{k_n + k}{k_n - k} , \quad (3)$$

where  $k$  is the (single) channel momentum and  $k_n$  pole locations in the complex  $k$  plane. The above formula is written down for any finite range potential, in  $s$  wave. Notice that the Eq. (3) automatically predicts a negative background contribution!

Near physical threshold, on the left hand side of Eq. (1), one can replace the physical amplitude by the one calculated using  $\chi$ PT upto certain powers of external momentum. On the right hand side one has a genuine parametrization form for resonance contributions. If expand both sides at, for example,  $\pi\pi$  scattering threshold, then one should be able to obtain useful relations between low energy constants of  $\chi$ PT and resonance parameters, without relying on any lagrangian models describing these resonances. There is however a difficulty in estimating the left hand cut integral on the *r.h.s.* of Eq. (1), in which both crossed channel  $\pi\pi$  cut and resonance exchanges contribute. Fortunately

**TABLE 1.** Summary of the different contributions  $T(0)$ ,  $t_0^{\text{tR}}$ ,  $t_0^{\text{sR}}$  to the scattering lengths at leading order in the  $m_\pi^2$  expansion.  $M$  and  $\Gamma$  represent the mass and width of a resonance, respectively. The unit of each amplitude is in  $m_\pi^2$ .

IJ	$T(0)$	$t_0^{\text{tR}}$	$t_0^{\text{sR}}$	$t_0^{\chi PT}$
11	$-\frac{1}{24\pi f^2}$	$\frac{4\Gamma_S}{9M_S^3} + \frac{2\Gamma_V}{M_V^3}$	$\frac{4\Gamma_V}{M_V^3}$	0
00	$-\frac{1}{32\pi f^2}$	$-\frac{4\Gamma_S}{3M_S^3} + \frac{36\Gamma_V}{M_V^3}$	$\frac{4\Gamma_S}{M_S^3}$	$\frac{7}{32\pi f^2}$
20	$\frac{1}{16\pi f^2}$	$-\frac{4\Gamma_S}{3M_S^3} - \frac{18\Gamma_V}{M_V^3}$	0	$-\frac{1}{16\pi f^2}$

in the large  $N_c$  limit the cut integrals can all be solved analytically, and one demonstrates that in the large  $N_c$  limit the Eq. (1) is equivalent to the expression of partial wave dispersion relation. [15] Useful relations can hence be obtained in this way as expected.

This matching at  $\pi\pi$  threshold has been done at  $O(p^4)$  level in Ref. [15], and extended to  $O(p^6)$  in Ref. [16]. Table 1 shows the matching results in IJ=00,11,20, three different channels, at  $O(p^2)$  level. In table 1  $T(0)$  takes the  $\chi PT$  value at  $s = 0$ ,  $t_0^{\text{tR}}$  represents the resonance contribution in the crossed channel,  $t_0^{\text{sR}}$  means the resonance contribution in the  $s$  channel. The sum of these three quantities equals to  $t_0^{\chi PT}$  denoting the  $\chi PT$  result on scattering amplitude at threshold. It is realized that the sum leads to the same equation in three different channels:

$$\frac{1}{16\pi f^2} = \frac{9\Gamma_V^{(0)}}{M_V^{(0)3}} + \frac{2\Gamma_S^{(0)}}{3M_S^{(0)3}}, \quad (4)$$

where superscript (0) means value in the chiral limit. The above equation is some times called the generalized KSFRF relation. [17] We learned an important lesson from the above discussion, that is *partial wave amplitudes remember crossing symmetry*. If the crossed channel effects (i.e., the second column in Table 1) were omitted incorrectly, then we would obtain three different relations. It was stated earlier that the [1,1] Padé approximation is equivalent to neglecting the crossed channel cut in the large  $N_c$  and chiral limit, hence it violates crossing symmetry.

The above discussion also helps in understanding why the Padé approximation works good in the IJ=11 channel: because the cut effects are very small in this channel, this property is sometimes named as vector meson dominance in the literature; why works not so good in the IJ=00 channel: because the left hand cut is non-negligible in this channel; and why it runs into disaster in the exotic IJ=20 channel: since in this channel there is only crossed channel effects! One may draw a further conclusion: the quality of Padé approximation relies on whether in the corresponding channel there is a single pole dominance.

If the  $\sigma$  pole does not maintain a proper  $N_c$  behavior, i.e.,  $M \sim O(1)$ ,  $\Gamma \sim O(1/N_c)$ , then it would be very difficult for the three channels to fulfil the constraints from crossing

symmetry simultaneously.

The Eq. (4) also explains, in one aspect, why the  $\sigma$  meson had been escaped from detection: its contribution to the *r.h.s.* is numerically tiny as compared with that of the vector meson. The vector meson plays a more prominent role than the scalar meson in most cases.

As stated in the begining of this talk, if one construct a unitary amplitude from perturbative amplitude calculated using chiral perturbation theory lagrangian, one finds a light and broad pole on the complex  $s$  plane. Since this pole is absent in the original chiral lagrangian, it is called ‘dynamically generated resonance’. [6] It is also found that the  $N_c$  trajectory of such generated dynamical  $\sigma$  pole has a non-typical behavior as comparing with that of a normal resonance, e.g., a  $\rho$  pole. This is viewed as further evidence to support the dynamical nature of the sigma pole. One may even relate such exotic behavior to the tetra quark state.

Despite of the problem a unitarization approximation may encounter as described in previously, the above phenomenon is itself interesting and worthy of a careful examination.

To clarify the meaning of ‘dynamically generated resonance’, we make a simple exercise by calculating everything in the toy  $O(N_f)$  linear sigma model. This model is exactly solvable, because when  $N_f$  is large Feynman diagrams that contribute are greatly reduced, excluding those associated with crossed channel dynamics. One can also pretend to solve it using [1,1] Padé approximant, after integrating out the sigma degree of freedom, at tree level. The latter simulates the calculation in reality.

In both cases it is found that the  $\sigma$  pole trajectory with respect to  $N_c$  in the  $O(N_f)$  model behaves in an exotic way: it does not like to fall down to the real axis but eventually it has to. [14]

Hence the bent structure of the sigma pole  $N_c$  trajectory in  $O(N)$  model and its variation, similar to what is found in a more realistic situation, does not at all suggest the  $\sigma$  pole itself is of dynamical origin.

We have shown the picture that a ‘dynamical resonance’ generated from a pure  $\chi$ PT lagrangian is neither necessary nor provides a better understanding to the physics underlined. We believe that the  $\sigma$  degree of freedom can be appropriately put in and described by a lagrangian respecting chiral symmetry. Hence a lagrangian with a linearly realized chiral symmetry is justified.

We have studied the scalar problem in the ENJ-L model (a model with linearly realized chiral symmetry) in  $SU_f(3)$  content. The scalar puzzle is of course not limited to the  $\sigma$  only. One has to examine the  $\kappa$ ,  $f_0(980)$ ,  $a_0(980)$  altogether in order to achieve a better understanding. Using a crude K-matrix unitarization scheme, we have shown that, the  $\sigma$ ,  $\kappa$  and the  $a_0(980)$  can be explained as members of one scalar nonet. [18] Despite of the crude approximation being made, this conclusion is not quite trivial and apparent. Remember that the three scalars, not only have distorted masses, but also very different widths. Also interestingly we found that the  $f_0(980)$  is not able to be included in the same nonet. The latter does not disprove our picture that the  $\sigma$  is accompanied by a whole octet, since the  $f_0(980)$  very likely owns a quite different physical explanation. [19, 20]

To conclude, we have found that all evidence accumulated so far are consistent with the picture that the  $f_0(600)$  resonance is nothing but the  $\sigma$  meson responsible for a

spontaneous breaking of linearly realized chiral symmetry, and be the chiral partner of the Nambu–Goldstone bosons.

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